

# Conditions and instability in $f(R)$ gravity with non-minimal coupling between matter and geometry\*

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In this paper on the basis of the generalized  $f(R)$  gravity model with arbitrary coupling between geometry and matter, four classes of  $f(R)$  gravity models with non-minimal coupling between geometry and matter will be studied. By means of conditions of power-law expansion and the equation of state of matter less than  $-1/3$ , the relationship among  $p$ ,  $w$  and  $n$ , the conditions and the candidate for late-time cosmic accelerated expansion will be discussed in the four classes of  $f(R)$  gravity models with non-minimal coupling. Furthermore, in order to keep considering models to be realistic ones, the DolgovCKawasaki instability will be investigated in each of them.

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## 1. Introduction

According to recent observational data sets[1–3], our current universe is flat and undergoing a phase of the accelerated expansion which started about five billion years ago. To explain this phenomena, a variety of models have been proposed which may be divided into three broad classes. First, it is possible that there is some undiscovered property in our existing model of gravity and matter that leads to acceleration at the present time. In thses scenarios, one might consider including the existence of a tiny cosmological constant and the possibility of the backreaction of cosmological perturbations.

Second is the idea that the Universe is dominated by an exotic component with large negative pressure, usually referred to as dark energy. The simplest form of dark energy is cosmological constant  $\Lambda$  which would encounter fine-tuning problem and coincidence problem. Other valid dark energy models are provided by scalar fields, such as: Quintessence[4, 5], which is introduced to solve the coincidence problem and characterized by the equation of state (EOS)  $w_{de}$  between -1 and -1/3 (namely,  $-1 < w_{de} < -1/3$ ); Phantom (ghost) field[6], which owns a negative kinetic energy and characterized by the EOS  $w_{de}$  less than -1 (namely,  $w_{de} < -1$ ); Tachyon field[7, 8] which can act as a source of dark energy depending upon the form of the tachyon potential, and so on. Other scenarios on dark energy include brane world[9], generalized Chaplygin gas[10], holographic dark energy[11], etc. Unfortunately, up to now a satisfactory answer to the question that what dark energy is and where it came from has not yet to be obtained.

Finally, eliminating the need of dark energy, one may consider modified theories of gravity in which the late-

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time cosmic accelerated expansion can be realized by an infrared modification. There are numerous ways to deviate from Einstein's theory of gravity. The most well-known alternative to General Relativity (GR) is scalar-tensor theory[12, 13]. There are still numerous proposals for modified theories of gravity in contemporary literature, such as DGP (Dvali- Gabadadze-Porrati) gravity[14], braneworld gravity[15], TeVeS (Tensor-Vector- Scalar)[16],  $f(R)$  theories of gravity[17], Einstein-Aether theory[18] and so on.

Among these theories,  $f(R)$  gravity is very competitive. Here  $f(R)$  is an arbitrary function of the Ricci scalar  $R$ . One can add any form of  $R$  in it, such as  $1/R$ [19] (the simplest one),  $\ln R$ [20], positive and negative powers of  $R$ [21], Gauss-Bonnet invariant[22], etc. The more general forms of  $f(R)$  can be considered including coupling between  $f(R)$  and scalar[23], multidimensional  $1/R$  theory[24] and so on. In  $f(R)$  theories of gravity, the expansion history of the universe is naturally explained by the fact that some gravitational terms which support the inflation at early-time universe, while other terms which cause the cosmic acceleration at late-time universe. It is worth stressing that considering some additional conditions, the early-time inflation and late-time acceleration can be unified by different role of gravitational terms relevant at small and at large curvature. However,  $f(R)$  gravity is not perfect because of containing a number of instabilities. For instance, the theory with  $1/R$  may develop the instability[25]. But by adding a term of  $R^2$  to this specific  $f(R)$  model, one can remove this instability[20, 21]. For more general forms of  $f(R)$ , the stability condition  $f'' \geq 0$  can be used to test  $f(R)$  gravity models[26].

Recently, a general model of  $f(R)$  gravity has been proposed in Ref.[27], which contains a non-minimal coupling between geometry and matter. This coupling term can be considered as a gravitational source to explain the current acceleration of the universe. The viability criteria for such a theory was recently discussed in Refs.[27, 28]. However, a more general model, in which the coupling style is arbitrary and the Lagrangian density of matter only appears in coupling term, has been proposed in Ref.[29] and it can represent the former case. The purpose of this paper is to discuss the conditions for late-time behaviour in  $f(R)$  gravity with non-minimal coupling between matter and geometry.

This paper is organized as follows. In the next section, for the general  $f(R)$  gravity models with arbitrary and non-minimal couplings, the field equations, the energy conditions and the DolgovCKawasaki instability will be given, respectively. In this class of models, the energy-momentum tensor of matter is generally not conserved due to the appearance of an extra force as mentioned in Ref.[27]. The conditions for late-time cosmic accelerated expansion and the instability in  $f(R)$  gravity with non-minimal coupling will be discussed in sections 3 and 4, respectively. Four classes of models will be taken into consideration in those two sections. Summary is given in the last section.

## 2. THE GENERAL $f(R)$ GRAVITY WITH COUPLING BETWEEN MATTER AND GEOMETRY

A more general action in  $f(R)$  gravity, in which the coupling style between matter and geometry is arbitrary and the Lagrangian density of matter only appears in coupling term, is given by

$$S = \int \left[ \frac{1}{2} f_1(R) + G(L_m) f_2(R) \right] \sqrt{-g} d^4x, \quad (1)$$

where we have chosen  $\kappa = 8\pi G = c = 1$ , which we shall adopt hereafter.  $f_i(R)$  ( $i = 1, 2$ ) and  $G(L_m)$  are arbitrary functions of the Ricci scalar  $R$  and the Lagrangian density of matter respectively. When  $f_2(R) = 1$  and  $G(L_m) = L_m$ , we obtain the general form of  $f(R)$  gravity with non-coupling between matter and geometry. Furthermore, by setting

$f_1(R) = R$ , action (1) can be reduced to the standard General Relativity (GR).

Varying the action (1) with respect to the metric  $g^{\mu\nu}$  yields the field equations

$$\begin{aligned} F_1(R)R_{\mu\nu} - \frac{1}{2}f_1(R)g_{\mu\nu} + (g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)F_1(R) &= -2G(L_m)F_2(R)R_{\mu\nu} \\ -2(g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)G(L_m)F_2(R) - f_2(R)[K(L_m)L_m - G(L_m)]g_{\mu\nu} & \\ + f_2(R)K(L_m)T_{\mu\nu}, & \end{aligned} \quad (2)$$

where  $\square = g^{\mu\nu}\nabla_\mu\nabla_\nu$ ,  $F_i(R) = df_i(R)/dR$  ( $i = 1, 2$ ) and  $K(L_m) = dG(L_m)/dL_m$ . The energy-momentum tensor of matter is defined as:

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}L_m)}{\delta g^{\mu\nu}}. \quad (3)$$

Assuming that the Lagrangian density of matter  $L_m$  only depends on the metric tensor components and not on its derivatives, we obtain

$$T_{\mu\nu} = L_m g_{\mu\nu} - 2 \frac{\partial L_m}{\partial g^{\mu\nu}}. \quad (4)$$

The trace of the field equations (2) reads

$$\begin{aligned} 3\square[F_1(R) + 2G(L_m)F_2(R)] + [F_1(R) + 2G(L_m)F_2(R)]R \\ - 2f_1(R) + 4f_2(R)[K(L_m)L_m - G(L_m)] = K(L_m)f_2(R)T, \end{aligned} \quad (5)$$

where  $T = T^\mu_\mu$ .

By taking the covariant divergence of Eq.(2) and using the mathematical identity  $\nabla^\mu[f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + (g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)f(R)] \equiv 0$  [29], here  $f'(R) = df/dR$ , we have

$$\nabla^\mu T_{\mu\nu} = 2\nabla^\mu \ln[f_2(R)K(L_m)] \frac{\partial L_m}{\partial g^{\mu\nu}}, \quad (6)$$

from which we see that the conservation of the energy-momentum tensor of matter is violated due to the coupling between matter and geometry. However, once the  $L_m$  is given, by choosing appropriate forms of  $G(L_m)$  and  $f_2(R)$ , one can construct, at least in principle, conservative model with arbitrary matter-geometry coupling.

In order to keep the energy density is positive and cannot flow faster than light, the generalized energy conditions, namely, the strong energy condition (SEC), the null energy condition (NEC), the weak energy condition (WEC) and the dominant energy condition (DEC), should be taken into consideration, which forms can be derived as follows (see Ref.[30] for more details):

$$\begin{aligned} \rho + 3p - \frac{1}{f_2 G'}[f_1 - (f_1' + 2Gf_2')R] + 3\frac{f_1''}{f_2 G'}(H\dot{R} + \ddot{R}) \\ + 3\frac{f_1'''}{f_2 G'}\dot{R}^2 + 6\frac{1}{f_2 G'}(G''\dot{L}_m^2 f_2' + \ddot{L}_m G' f_2' + 2f_2''\dot{R}G'\dot{L}_m \\ + f_2''' \dot{R}^2 G + f_2'' \ddot{R}G) + 6\frac{H}{f_2 G'}(G'\dot{L}_m f_2' + f_2'' \dot{R}G) \\ + \frac{2}{G'}(G'\dot{L}_m - G) \geq 0, \quad (SEC) \end{aligned} \quad (7)$$

$$\begin{aligned} \rho + p + (H\dot{R} + \ddot{R})\frac{f_1''}{f_2 G'} + \frac{f_1'''}{f_2 G'}\dot{R}^2 + \frac{2}{f_2 G'}(G''\dot{L}_m^2 f_2' + \\ \ddot{L}_m G' f_2' + 2f_2''\dot{R}G'\dot{L}_m + f_2''' \dot{R}^2 G + f_2'' \ddot{R}G) - \\ \frac{2H}{f_2 G'}(G'\dot{L}_m f_2' + f_2'' \dot{R}G) \geq 0, \quad (NEC) \end{aligned} \quad (8)$$

$$\begin{aligned}
& \rho - p + \frac{1}{f_2 G'} [f_1 - (f_1' + 2G f_2') R] - (5H \dot{R} + \ddot{R}) \frac{f_1''}{f_2 G'} - \\
& \frac{f_1'''}{f_2 G'} \dot{R}^2 - \frac{2}{f_2 G'} (G'' L_m^2 f_2' + \ddot{L}_m G' f_2' + 2f_2'' \dot{R} G' L_m + \\
& f_2''' \dot{R}^2 G + f_2'' \ddot{R} G) - \frac{10H}{f_2 G'} (G' L_m f_2' + f_2'' \dot{R} G) - \frac{2}{G'} \\
& (G' L_m - G) \geq 0, \quad (DEC)
\end{aligned} \tag{9}$$

$$\begin{aligned}
& \rho + \frac{1}{2f_2 G'} [f_1 - (f_1' + 2G f_2') R] - 3H \dot{R} \frac{f_1''}{f_2 G'} - 6H \frac{1}{f_2 G'} \\
& (G' L_m f_2' + f_2'' \dot{R} G) - \frac{1}{G'} (G' L_m - G) \geq 0, \quad (WEC)
\end{aligned} \tag{10}$$

where the dot denotes differentiation with respect to cosmic time. Moreover, by using condition  $f_1''(R) + 2G(L_m)f_2''(R) \geq 0$ , one can test the Dolgov-Kawasaki instabilities for this class of models.

When  $G(L_m) = L_m$  and rescales the function  $f_2(R)$  as  $1 + \lambda f_2(R)$ , the action (1) and the field equations (2) can be changed into

$$S = \int \left\{ \frac{1}{2} f_1(R) + [1 + \lambda f_2(R)] L_m \right\} \sqrt{-g} d^4x, \tag{11}$$

$$\begin{aligned}
& F_1(R) R_{\mu\nu} - \frac{1}{2} f_1(R) g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) F_1(R) = -2\lambda F_2(R) L_m R_{\mu\nu} \\
& + 2\lambda (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) L_m F_2(R) + [1 + \lambda f_2(R)] T_{\mu\nu}.
\end{aligned} \tag{12}$$

Above expressions are just the action and the field equations in  $f(R)$  gravity with non-minimal coupling between geometry and matter. Moreover, by means of the generalized Bianchi identities  $\nabla^\mu G_{\mu\nu} = 0$  (here,  $G_{\mu\nu}$  is the Einstein tensor), Eq.(6) can be given as:

$$\nabla^\mu T_{\mu\nu} = \frac{\lambda F_2}{1 + \lambda f_2} [g_{\mu\nu} L_m - T_{\mu\nu}] \nabla^\mu R. \tag{13}$$

It follows that the non-minimal coupling term results in a non-trivial exchange of energy and momentum between geometry and matter[31, 32]. Note that according to Eq.(13), the conservation of the energy-momentum tensor can be verified if  $f_2(R)$  is a constant or the Lagrangian density of matter is not an explicit function of the metric.

### 3. THE CONDITIONS FOR LATE-TIME COSMIC ACCELERATED EXPANSION IN $f(R)$ GRAVITY WITH NON-MINIMAL COUPLING

In the following, we focus on the conditions for late-time cosmic accelerated expansion in  $f(R)$  gravity with non-minimal coupling between geometry and matter. In this section, the form of the action is taken to be Eq.(11) and, for simplicity, we consider  $L_m$  is opposite to the energy density of perfect fluid[32], i.e.,

$$L_m = -\rho = -\rho_0 a^{-3(1+w)}, \tag{14}$$

where  $w$  is the equation of state of perfect fluid and is assumed to be a constant. The energy-momentum tensor is taken as:

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}, \tag{15}$$

where  $\rho$  and  $p$  denote the energy density and the pressure respectively. The form of the FRW metric is chosen as

$$ds^2 = -dt^2 + a^2(t)dX_3^2, \quad (16)$$

where  $a(t)$  is the scale factor and  $dX_3^2$  contains the spacial part of the metric. Using this metric, we can obtain  $R = 6(2H^2 + \dot{H})$ , where  $H = \dot{a}(t)/a(t)$  is the Hubble expansion parameter.

It is known that under the conditions either power-law expansion or the equation of state of matter less than  $-1/3$ , late-time cosmic accelerated expansion occurs. To exemplify how to use these conditions to realize the phase of accelerating expansion in  $f(R)$  gravity with non-minimal coupling, Firstly, we concentrate on two simple classes of models.

(1) Let

$$f_1(R) = R, \quad f_2(R) = -AR^{-n} + BR^2, \quad (17)$$

where  $A$  and  $B$  are arbitrary constants. Then, Eq.(12) becomes into

$$3H^2 = -\rho_0 a^{-3(1+w)} [1 + 6\lambda(H^2 + \dot{H})(24BH^2 + 12B\dot{H} + An(12H^2 + 6\dot{H})^{-1-n}) + \lambda(36B(2H^2 + \dot{H})^2 - A(12H^2 + 6\dot{H})^{-n})]. \quad (18)$$

We assume the solution of Eq.(18) is  $a = a_0 t^p$  and then have  $H = \frac{p}{t}$ ,  $\dot{H} = -\frac{p}{t^2}$ . Substituting these relations into Eq.(18), we find there are three kinds of possible relationships among  $p$ ,  $w$  and  $n$ , namely,  $p = \frac{2(n+1)}{3(1+w)}$ ,  $p = \frac{2}{3(1+w)}$  and  $p = \frac{-2}{3(1+w)}$ . Under the condition of power-law expansion (i.e.,  $p > 1$ ), the corresponding regions of  $w$  are  $w < \frac{2(n+1)}{3} - 1$  for  $p = \frac{2(n+1)}{3(1+w)}$ ,  $w < -1/3$  for  $p = \frac{2}{3(1+w)}$  and  $w < -5/3$  for  $p = \frac{-2}{3(1+w)}$ , respectively. Furthermore, by considering the equation of state of matter less than  $-1/3$  (i.e.,  $w < -1/3$ ), we can obtain that when  $p = \frac{2(n+1)}{3(1+w)}$ , the range of parameter  $n$  is  $n \leq 0$  and  $n \neq -1$ . It is easy to see that there is no constraint on  $n$  when cases  $p = \frac{2}{3(1+w)}$  and  $p = \frac{-2}{3(1+w)}$ . For the case  $p = \frac{2(n+1)}{3(1+w)}$ , the effective quintessence regime ( $-1 < w < -1/3$ ) emerges when  $-1 < n \leq 0$  and the effective phantom regime ( $w < -1$ ) emerges when  $n < -1$ . The candidate for late-time cosmic accelerated expansion can be either the effective quintessence or the effective phantom, when  $p = \frac{2}{3(1+w)}$ .

(2) Another choice for functions  $f_1(R)$  and  $f_2(R)$  are

$$f_1(R) = R, \quad f_2(R) = \frac{c_1 R^n}{c_2 R^n + 1}, \quad (19)$$

where  $c_1$  and  $c_2$  are constants. Then the FRW equation is changed into

$$3H^2 = -\rho_0 a^{-3(1+w)} [1 + \frac{6^n c_1 n \lambda (H^2 + \dot{H})(2H^2 + \dot{H})^{-1+n}}{[1 + 6^n c_2 (2H^2 + \dot{H})^n]^2} + \frac{6^n c_1 \lambda (2H^2 + \dot{H})^n}{1 + 6^n c_2 (2H^2 + \dot{H})^n}]. \quad (20)$$

By calculations and analysis, the relationship among  $p$ ,  $w$  and  $n$ , condition and candidate for late-time cosmic accelerated expansion are shown in Table I.

It is worth stressing that the de Sitter stage is impossible in both models because there is a scale factor  $a$  in the FRW equation. If the function  $f_2(R)$  vanishes, the standard FRW equation would be reproduced. Above forms of  $f_2(R)$  have been discussed in Refs.[21, 33]. Next, we focus on other two complicated models.

(3) Following Ref.[21], let us take the following explicit choice for functions  $f_1(R)$  and  $f_2(R)$  as:

$$f_1(R) = R - AR^{-n} + BR^2, \quad f_2(R) = -AR^{-n} + BR^2, \quad (21)$$

Relationship	Condition	Candidate	
		The effective quintessence	The effective phantom
$p = \frac{2(1-n)}{3(1+w)}$	$n \geq 0$ and $n \neq 1$	$0 \leq n < 1$	$n > 1$
$p = \frac{2(1-2n)}{3(1+w)}$	$n \geq 0$ and $n \neq 1/2$	$0 \leq n < -1/2$	$n > 1/2$
$p = \frac{2}{3(1+w)}$	$w < -1/3$	All $n$	All $n$

TABLE I: The relationship among  $p$ ,  $w$  and  $n$ , condition and candidate for late-time cosmic accelerated expansion in case  $f_1(R) = R$ ,  $f_2(R) = \frac{c_1 R^n}{c_2 R^{n+1}}$ .

where  $A$  and  $B$  are arbitrary constants. Then the equation (12) can be expressed as:

$$\begin{aligned} \frac{1}{2}\{12H^2 + 6\dot{H} + 36B(2H^2 + \dot{H})^2 - A(12H^2 + 6\dot{H})^{-n} - 6(H^2 + \dot{H})[1 + 24BH^2 + \\ 12B\dot{H} + An(12H^2 + 6\dot{H})^{-1-n}]\} = -\rho_0 a^{-3(1+w)}\{1 + 6\lambda(H^2 + \dot{H})[24BH^2 + 12B\dot{H} + \\ An(12H^2 + 6\dot{H})^{-1-n}] + \lambda[36B(2H^2 + \dot{H})^2 - A(12H^2 + 6\dot{H})^{-n}]\}. \end{aligned} \quad (22)$$

By the same method as above, we find there are four kinds of possible relationships among  $p$ ,  $w$  and  $n$ , i.e.,  $p = \frac{4}{3(1+w)}$ ,  $p = -\frac{2n}{4+3w}$ ,  $p = \frac{2}{3(1+w)}$  and  $p = -2n$ . Under conditions of power-law expansion and the equation of state of matter less than  $-1/3$ , above results are turned into  $w < 1/3$  for  $p = \frac{4}{3(1+w)}$ ,  $n \geq -3/2$  for  $p = -\frac{2n}{4+3w}$ ,  $w < -1/3$  for  $p = \frac{2}{3(1+w)}$  and  $n < -1/2$  for  $p = -2n$ , respectively. It is clear that the candidate for late-time cosmic accelerated expansion is among dust, the effective quintessence and the effective phantom when  $p = \frac{4}{3(1+w)}$ . But except dust when  $p = \frac{2}{3(1+w)}$ . Note that there is no constraint on  $n$  in both cases. For the case  $p = -\frac{2n}{4+3w}$ , either the effective quintessence regime emerges when  $-3/2 \leq n < -1/2$  or the effective phantom regime emerges when  $n \geq -1/2$  and  $n \neq 0$ . Obviously, the late-time cosmic accelerated expansion is independent of matter when  $p = -2n$ .

(4) Another choice for functions  $f_1(R)$  and  $f_2(R)$  are

$$f_1(R) = R + \frac{c_1 R^n}{c_2 R^{n+1}}, \quad f_2(R) = \frac{c_1 R^n}{c_2 R^{n+1}}, \quad (23)$$

where  $c_1$  and  $c_2$  are constants. In this case, the FRW equation can be given as:

$$\begin{aligned} \frac{1}{2}[12H^2 + 6\dot{H} + \frac{6^n c_1 (2H^2 + \dot{H})^n}{1+6^n c_2 (2H^2 + \dot{H})^n} - 6(H^2 + \dot{H})(1 + \frac{6^{-1+n} c_1 n (2H^2 + \dot{H})^{-1+n}}{[1+6^n c_2 (2H^2 + \dot{H})^n]^2})] = \\ -\rho_0 a^{-3(1+w)}[1 + \frac{6^n c_1 n \lambda (H^2 + \dot{H})(2H^2 + \dot{H})^{-1+n}}{[1+6^n c_2 (2H^2 + \dot{H})^n]^2} + \frac{6^n c_1 \lambda (2H^2 + \dot{H})^n}{1+6^n c_2 (2H^2 + \dot{H})^n}]. \end{aligned} \quad (24)$$

The corresponding relationship among  $p$ ,  $w$  and  $n$ , condition and candidate for late-time cosmic accelerated expansion are shown in Table II.

Obviously, the de Sitter stage is also impossible in both models and the reason is as the same as the ones in above two simple cases. If the function  $f_2(R)$  vanishes, the modified gravity with non-coupling can be reproduced. Above forms of  $f_1(R)$  have been discussed in Refs.[21, 34].

From the above discussions, it is easy to see that the results in complicated models are more interesting than the simple ones.

For the four models mentioned above, the transition from matter- dominated phase to the acceleration phase discussions without non-minimal coupling have been made in Ref.[35]could be realized as follows. Since the Hubble parameter can be expressed as  $H = p/t$ , Ricci scalar  $R$  turns into  $R = 6p(2p - 1)/t^2$ . If  $0 < p < 1$ , the early universe

Relationship	Condition	Candidate	
		The effective quintessence	The effective phantom
$p = \frac{2}{3(1+w)}$	$w < -1/3$	All $n$	All $n$
$p = \frac{4n+2}{3(1+w)}$	$n \leq 0$ and $n \neq -1/2$	$-1/2 < n \leq 0$	$n < -1/2$
$p = \frac{2(n+1)}{3(1+w)}$	$n \leq 0$ and $n \neq -1$	$-1 < n \leq 0$	$n < -1$
$p = \frac{2n}{3(1+w)}$	$n \leq 1$ and $n \neq 0$	$0 < n \leq 1$	$n < 0$
$p = \frac{4n}{3(1+w)}$	$n \leq 1/2$ and $n \neq 0$	$0 < n \leq 1/2$	$n < 0$

TABLE II: The relationship among  $p$ ,  $w$  and  $n$ , condition and candidate for late-time cosmic accelerated expansion in case  $f_1(R) = R + \frac{c_1 R^n}{c_2 R^n + 1}$ ,  $f_2(R) = \frac{c_1 R^n}{c_2 R^n + 1}$ .

is in deceleration phase, which corresponds to matter-dominated phase with  $p = 2/3$ , and if  $p > 1$ , the late universe is in acceleration phase.

#### 4. THE INSTABILITY OF $f(R)$ GRAVITY WITH NON-MINIMAL COUPLING

A viable modified gravity model must pass Newton law, solar system test and instability conditions[21, 36]. There are in principle several kinds of instabilities to consider[37]. DolgovCKawasaki instability[38] is one of them. Below, we will focus on this instability. According to Ref.[30], the DolgovCKawasaki criterion in  $f(R)$  gravity with non-minimal coupling between matter and geometry is

$$f_1''(R) + 2\lambda L_m f_2''(R) \geq 0 \quad (25)$$

where  $\lambda$  is a constant and  $L_m$  is the Lagrangian density of matter. For simplicity assuming  $A$ ,  $B$ ,  $C_1$ ,  $C_2$  are positive constants, the DolgovCKawasaki criterions for above four discussed models are as follows:

$$n \leq 0 \text{ or } n \leq -1, \quad \text{for model 1 and model 3,} \quad (26)$$

$$n \leq \frac{1 + C_2 R^n}{1 - C_2 R^n}, \quad \text{for model 2 and model 4} \quad (27)$$

By means of analysis of model 1, model 3 and Eq.(26), we find that they could be realistic candidates for late-time cosmic accelerated expansion without DolgovCKawasaki instability. By taking Table I, Table II and Eq.(27) into consideration, model 2 and model 4 would be realistic candidates if  $1 + C_2 R^n / 1 - C_2 R^n > 0$  and  $1 + C_2 R^n / 1 - C_2 R^n \leq 1$  respectively. Otherwise there is no interesting in model 2. Furthermore, the DolgovCKawasaki instability will be emerged in model 4, if  $1 + C_2 R^n / 1 - C_2 R^n < 1/2$  (i.e.  $1/2 \leq n \leq 1$ ) for case  $p = 2n/(1+w)$ ,  $0 \leq n \leq 1$  for case  $p = 2n/(1+w)$  and  $0 \leq n \leq 1/2$  for case  $p = 4n/(1+w)$ , respectively.

#### 5. SUMMARY

Up to now, we have discussed the conditions for late-time cosmic accelerated expansion and the DolgovCKawasaki instability in  $f(R)$  gravity with non-minimal coupling between geometry and matter. For simplicity, we chose the

form of the Lagrangian density of matter as opposite to the energy density of perfect fluid. The relationship among  $p$ ,  $w$  and  $n$  has been given in each class of models. By using the conditions of power-law accelerated expansion, the equation of state of matter less than  $-1/3$  and the DolgovKawasaki criterion, the range of the parameter  $n$  is concretely constrained. Either the effective quintessence regime or the effective phantom regime would emerge by choosing  $n$  properly. It is easy to see that the results in complicated models are more interesting than the simple ones. It is demonstrated that the de Sitter stage would not realize in all considering models because there is a scale factor  $a$  in FRW equation. Essentially, this is due to the special choice of the Lagrangian density of matter. Other forms of the Lagrangian density of matter could be considered in the similar fashion of non-minimal gravitational coupling.

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